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Abstract. A method is developed for the calculation of the matrix element for any nuclear reaction in which n nucleons are transferred from the projectile to the target nucleus. As an example, the matrix element for the four-nucleon transfer reaction is derived in the Born approximation. Comparison with the experimental angular distribution for the reaction $^{12}C(d, {}^6Li)^8Be$ is carried out in both the diffraction and the plane wave models and the results are compared with the experimental data.

1. Introduction

The theory of direct reactions was applied successfully to one (Butler 1951), two (El-Nadi 1957, 1960, El-Nadi and El-Khishin 1959, El-Nadi and Sherif 1961, 1962, Newns 1960, Glendenning 1960, 1965, Yoshida 1962), and three-nucleon transfer reactions (El-Nadi and Sherif 1964, El-Nadi *et al.* 1967). In the treatments of multiple-nucleon transfers of two or more nucleons the nucleons may be transferred as a cluster (Honda *et al.* 1964) or as independent particles. In such cases the single-cluster stripping will be similar to the single-nucleon stripping process. One may wonder whether, in such multi-nucleon transfer processes, there will be a difference in the results by considering the transferred nucleons as a single cluster or as separate particles. In the present work a general procedure is outlined for the derivation of the matrix element for the n-nucleon transfer reaction. The transferred particles are considered independently. The shell model is used for the description of the nuclear states, and harmonic oscillator wave functions are used for the single-particle states. These assumptions enabled one to use the parentage coefficients for separating the wave function of the stripped nucleons from that of N nucleons, and to make use of the Talmi transformation techniques to separate the relative motion from the centre-of-mass motion (Moshinsky 1959, Smirnov 1961).

2. Formulation

Let $\Psi_{MJt\tau}(nlj)^N$ be the wave function of a nucleus consisting of N nucleons in its incomplete shell (nlj). By the fractional parentage coefficients one may separate from this wave function the wave function of n nucleons:

$$\begin{aligned}\Psi_{JMt\tau}(nlj)^N = \sum \begin{Bmatrix} \alpha_1\alpha_n J_1 J_n t_1 \\ t_n M_{J_1} M_{J_n} \tau_1 \tau_n \end{Bmatrix} (j^{N-n}(\alpha_1)J_1, j^n(\alpha_n)J_n;\ J|\}j^N(\alpha)J) \\ \times (J_1 M_{J_1}, J_n M_{J_n}|JM)(t_1\tau_1,\ t_n\tau_n|t\tau)\Psi_{J_1 M J_1 t_1 \tau_1}(nlj)^{N-n} \\ \times \Psi_{J_n M_{J_n} t_n \tau_n}(nlj)^n \qquad (1)\end{aligned}$$

where $(\ |\}\ )$ is the fractional parentage coefficient. Then, using the convenient transformation brackets, one may change the coupling scheme for the wave function $\Psi_{J_n M_{J_n} t_n \tau_n}(nlj)^n$ from jj coupling into lS coupling:

$$\begin{aligned}\Psi_{J_n M_{J_n} t_n \tau_n}(nlj)^n = \sum \begin{pmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ \mathscr{L}_2 & S_2 & J_2 \end{pmatrix} \begin{pmatrix} \mathscr{L}_2 & S_2 & J_2 \\ l_3 & \frac{1}{2} & j_3 \\ \mathscr{L}_3 & S_3 & J_3 \end{pmatrix} \cdots \begin{pmatrix} \mathscr{L}_{n-1} & S_{n-1} & J_{n-1} \\ l_n & \frac{1}{2} & j_n \\ \mathscr{L}_n & S_n & J_n \end{pmatrix} \\ \times (\mathscr{L}_n \mathscr{M}_{\mathscr{L}_n}, S_n m_{S_n}|J_n M_{J_n})\Psi_{\mathscr{L}_n \mathscr{M}_{\mathscr{L}_n}}(nl)^n \chi_{S_n m_{S_n}} \Omega_{t_n \tau_n} \qquad (2)\end{aligned}$$

where

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = ([c][f][g][h])^{1/2} \begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix}$$

$\begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix}$ is the $9j$ symbol and $[a] = 2a+1$.

Now, by the Talmi-transformation technique, one may change the wave function describing the single-particle motion into a function of the relative and centre-of-mass motion. Using Talmi coefficients of the first kind (Moshinsky 1959, Smirnov 1961) the single-particle wave functions of the first two nucleons are converted into the wave function of the relative distance between the two nucleons and the wave function of their centre of mass. By suitable angular momentum transformations one may then couple the orbital momentum of the centre-of-mass motion of the first two nucleons with the orbital momentum of the third, using Talmi transformations of the second kind leading to the wave function describing the relative motion between the centre of mass of the first two nucleons and the third nucleon, multiplied by the wave function describing the centre-of-mass motion of the three nucleons. This process is continued to the n nucleons, and one may finally write

$$\begin{aligned} \Psi_{\mathscr{L}_n \mathscr{M}_{\mathscr{L}_n}}(nl)^n = \sum \prod_{i=2}^{n} & \langle N_{i'} L_{i'},\ n_{i'-1,i} l_{i'-1,i};\ g_i | i'-1,\ i | N_{i'-1} L_{i'-1},\ n_i l_i;\ g_i \rangle \\ & \times a \begin{pmatrix} l_1 l_2(\mathscr{L}_2) l_3(\mathscr{L}_3) & \dots & \mathscr{L}_n \\ L_{n'} l_{n'-1,n}(g_n) & \dots & \mathscr{L}_n \end{pmatrix} (l_{12} m_{12},\ G_n m_{G_n} | \mathscr{L}_n \mathscr{M}_{\mathscr{L}_n}) \dots \\ & \times (L_{n'} M_{n'},\ l_{n'-1,n} m_{n'-1,n} | g_n m_{g_n}) \Psi_{n_{i'-1,i} l_{i'-1,i} m_{i,-1,i}}(r_{i-1,i}) \\ & \times \Psi_{N_{n'} L_{n'} M_{n'}} \end{aligned} \tag{3}$$

where $\langle \| \rangle$ is the Talmi transformation coefficient and $a \begin{pmatrix} l_1 l_2(\mathscr{L}_2) \dots \mathscr{L}_n \\ L_{n'} l_{n'-1} \dots \mathscr{L}_n \end{pmatrix}$ represents all the transformation brackets required to make the orbital momentum $L_{i'}$ couple with l_{i+1}; $\Psi_{n_{i'-1,i} l_{i'-1,i} m_{i'-1,i}}$ is the wave function describing the relative motion between the centre of mass of $i'-1$ nucleons and the ith nucleons, and $\Psi_{N_{n'} L_{n'} M_{n'}}(\boldsymbol{R}_n)$ is the wave function describing the motion of the centre of mass of the n nucleons. The a coefficients and the Clebsch–Gordan coefficients can be written after writing the Talmi transformation brackets.

As an illustration of equation (3) one may consider the cases of $n = 2$ and 3. For $n = 2$, $i = 2$ only, and equation (3) gives

$$\begin{aligned} \Psi_{\mathscr{L}_2 \mathscr{M}_{\mathscr{L}_2}}(nl)^2 = \sum & \langle N_2 L_2,\ n_{12} l_{12};\ \mathscr{L}_2 | 1,\ 1 | n_1 l_1,\ n_2 l_2;\ \mathscr{L}_2 \rangle \\ & \times (L_2 M_2,\ l_{12} m_{12} | \mathscr{L}_2 \mathscr{M}_{\mathscr{L}_2}) \Psi_{n_{12} l_{12} m_{12}}(\boldsymbol{r}_{12}) \Psi_{N_2 L_2 M_2}(\boldsymbol{R}_2). \end{aligned}$$

For $n = 3$, $i = 2$ and 3, and one obtains

$$\begin{aligned} \Psi_{\mathscr{L}_3 \mathscr{M}_{\mathscr{L}_3}}(nl)^3 = \sum & \langle N_2 L_2,\ n_{12} l_{12};\ \mathscr{L}_2 | 1,\ 1 | n_1 l_1,\ n_2 l_2;\ \mathscr{L}_2 \rangle \\ & \times \langle N_{3'} L_{3'},\ n_{2'3} l_{2'3};\ g_3 | 2,\ 1 | N_2 L_2,\ n_3 l_3;\ g_3 \rangle \begin{pmatrix} L_2 & l_{12} & \mathscr{L}_2 \\ \mathscr{L}_3 & l_3 & g_3 \end{pmatrix} \\ & \times (L_{3'} M_{3'},\ l_{2'3} m_{2'3} | g_3 m_{g_3}) (g_3 m_{g_3},\ l_{12} m_{12} | \mathscr{L}_3 \mathscr{M}_{\mathscr{L}_3}) \\ & \times \Psi_{n_{12} l_{12} m_{12}}(\boldsymbol{r}_{12}) \Psi_{n_{2'3} l_{2'3} m_{2'3}}(\boldsymbol{r}_{2'3}) \Psi_{N_{3'} L_{3'} M_{3'}}(\boldsymbol{R}_3) \end{aligned}$$

where

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = (-)^{a+b+d+e} \begin{Bmatrix} a & b & c \\ d & e & f \end{Bmatrix}$$

and where $\begin{Bmatrix} a & b & c \\ d & e & f \end{Bmatrix}$ is the $6j$ symbol.

Now, the transition amplitude for the reaction A(a, b)B, in which n nucleons are transferred from the projectile 'a' to the target 'A' to form 'B', in the distorted wave Born approximation (DWBA), may be written as

$$T_{\mathrm{fi}} = \langle \chi_{\boldsymbol{k}_{\mathrm{f}}}^{(-)}(\boldsymbol{r}_{\mathrm{f}}) \psi_{\mathrm{B}} \psi_{\mathrm{b}} | V_{\mathrm{f}} - U_{\mathrm{f}} | \chi_{\boldsymbol{k}_{\mathrm{i}}}^{(+)}(\boldsymbol{r}_{\mathrm{i}}) \psi_{\mathrm{A}} \psi_{\mathrm{a}} \rangle. \tag{4}$$

V_{f} is the interaction potential in the final reaction channel, and U_{f} is the b–B optical model potential.

Substituting from equations (1), (2) and (3) into equation (4), one obtains for the reaction A(a,b)B

$$\begin{aligned} T_{\mathrm{fi}} = \sum \mathscr{S}(\mathrm{a};\, \mathrm{b},\, \mathrm{c}) \mathscr{S}^*(\mathrm{B};\, \mathrm{A},\, \mathrm{c}) & (l_{12} m_{12},\, G_n m_{G_n} | \mathscr{L}_2 \mathscr{M}_{\mathscr{L}_n}) \\ & \ldots (L_{n'} M_{n'},\, l_{n'-1,n} m_{n'-1\, n} | g_n m_{g_n}) (l_{12}' m_{12}',\, G_n' m_{G_n}' | \mathscr{L}_n' \mathscr{M}_{\mathscr{L}_n'}) \\ & \ldots (L_n' M_n',\, l_{n'-1,n}' m_{n'-1,n}' | g_n' m_{g_n}') (J_{\mathrm{b}} m_{\mathrm{b}},\, J_{\mathrm{c}} m_{\mathrm{c}} | J_{\mathrm{a}} m_{\mathrm{a}}) \\ & \times (J_{\mathrm{A}} M_{\mathrm{A}},\, J_{\mathrm{c}} m_{\mathrm{c}} | J_{\mathrm{B}} M_{\mathrm{B}}) \mathscr{T} \end{aligned} \tag{5}$$

where $\mathscr{S}$ is the spectroscopic factor containing all the parentage coefficients, Talmi transformation and the angular momentum transformation brackets (Abul-Magd *et al.* 1967), and $\mathscr{T}$ is given by

$$\begin{aligned} \mathscr{T} = \langle \chi_{\boldsymbol{k}_{\mathrm{f}}}^{(-)}(\boldsymbol{r}_{\mathrm{f}}) \prod_{j=2}^{n} [\Psi_{n'\, _{-1,j} l'_{j'-1,j} m'_{j'-1,j}}(\boldsymbol{r}_{j'-1,j})] \\ \times \Psi_{N'_{n'} L'_{n'} M'_{n'}}(\boldsymbol{R}_n) | V_{\mathrm{f}} - U_{\mathrm{f}} | \chi_{\boldsymbol{k}_{\mathrm{i}}}^{(+)}(\boldsymbol{r}_{\mathrm{i}}) \prod_{i=2}^{n} \\ \times [\Psi_{n_{i-1,i} l_{i'-1,i} m_{i'-1,i}}(\boldsymbol{r}_{i'-1,i})] \Psi_{N_{n'} L_{n'} M_{n'}}(\boldsymbol{\rho}_n) \rangle. \end{aligned} \tag{6}$$

If we write the interaction potential as $V_{\mathrm{f}} = V_{\mathrm{bc}} + V_{\mathrm{bA}}$, the latter part is approximately cancelled by the potential U_{f}. The residual potential V_{bc} may be expressed as $\Sigma_{i=1}^{0} V_{i\mathrm{b}}$, i.e. as a sum of nucleon–nucleon potentials. As a result of the antisymmetry of the wave functions for the particles b and c, each of these terms will therefore give equal contributions, and hence

$$V_{\mathrm{bc}} = n V_{n\mathrm{b}}(r_{n\mathrm{b}}).$$

3. Four-nucleon transfer reaction

For the case of the four-nucleon transfer reaction, $\mathscr{S}(\alpha;\, \beta,\, \gamma)$ will be given by

$$\begin{aligned} \mathscr{S}(\alpha;\, \beta,\, \gamma) = (j^{N-4}(\alpha_1) J_\beta,\, j^4(\alpha_2) J_\gamma;\, J_\alpha | \} j^N(\alpha) J_\alpha) \\ \times \langle N_2 L_2,\, n_{12} l_{12};\, \mathscr{L}_2 | 1,\, 1 | n_1 l_1,\, n_2 l_2;\, \mathscr{L}_2 \rangle \\ \times \langle N_{3'} L_{3'},\, n_{2'3} l_{2'3};\, g_3 | 2,\, 1 | N_{2'} L_{2'},\, n_3 l_3;\, g_3 \rangle \\ \times \langle N_4 L_4,\, n_{3'4} l_{3'4};\, g_4 | 3,\, 1 | N_{3'} L_{3'},\, n_4 l_4;\, g_4 \rangle \begin{pmatrix} L_2 & l_{12} & \mathscr{L}_2 \\ \mathscr{L}_3 & l_3 & g_3 \end{pmatrix} \\ \times \begin{pmatrix} g_3 & l_{12} & \mathscr{L}_3 \\ \mathscr{L}_4 & l_4 & G_4 \end{pmatrix} \begin{pmatrix} L_{3'} & l_{2'3} & g_3 \\ G_4 & l_4 & g_4 \end{pmatrix} \end{aligned} \tag{7}$$

and $\mathscr{T}$ is given by

$$4\langle\chi_{k_f}^{(-)}\left(\rho_4+\frac{A}{A+4}R_4\right)\Psi^*_{n_{12}'l_{12}'m_{12}'}(r_{12})\Psi^*_{n'_{2'3}l'_{2'3}m'_{2'3}}(r_{2'3})$$
$$\times\Psi^*_{n'_{3'4}l'_{3'4}m'_{3'4}}(r_{3'4})\Psi^*_{N'_4L'_4M'_4}(R_4)|V(\rho_{4b})|\chi_{k_i}^{(+)}\left(R_4+\frac{b}{b+4}\rho_4\right)$$
$$\times\Psi^*_{n_{12}l_{12}m_{12}}(r_{12})\Psi^*_{n_{2'3}l_{2'3}m_{2'3}}(r_{2'3})\Psi^*_{n_{3'4}l_{3'4}m_{3'4}}(r_{3'4})\Psi^*_{N_4L_4M_4}(\rho_4)\rangle. \tag{8}$$

Using the Gaussian form for the potential $V_{b4}(\rho_{4b})$, and the oscillator form for the wave function Ψ, one obtains

$$\begin{aligned}\mathscr{T} = {} & \delta_{n_{12}n_{12}'}\delta_{l_{12}l_{12}'}\delta_{m_{12}m_{12}'}\delta_{n_{2'3}n'_{2'3}}\delta_{l_{2'3}l'_{2'3}}\delta_{m_{2'3}m'_{2'3}}4V_0\\
& \times\sum g_{l'_{3'4}}g_{l_{3'4}}(l_{3'4}-\lambda'(m'_{3'4}-m_{\lambda'}),\lambda'm_{\lambda'}|l'_{3'4}m'_{3'4'})\\
& \times(l_{3'4}-\lambda(m_{3'4}-m_\lambda),\lambda m_\lambda|l_{3'4}m_{3'4})(-\sigma)^{\lambda+\lambda'}\mathscr{N}_{l_{3'4}}\mathscr{N}_{l'_{3'4}}\\
& \times\int \mathrm{d}R\,\mathrm{d}\rho\,\mathrm{d}v\,\chi_{k_f}^{(-)*}\left(\rho_4+\frac{A}{A+4}R_4\right)\chi_{k_i}^{(+)}\left(R+\frac{b}{b+4}\rho\right)\Psi^*_{N'_4L'_4M'_4}\\
& \times\Psi^*_{N_4L_4M_4}(\rho)\exp(-\epsilon v^2-\delta R^2)\,Y_{\lambda m_\lambda}(\hat{R})\,Y^*_{\lambda' m_{\lambda'}}(\hat{R})\,Y_{l_{3'4}-\lambda,(m_{3'4}-m_\lambda)}(\hat{v})\\
& \times Y^*_{l'_{3'4}-\lambda',(m'_{3'4}-m_{\lambda'})}(\hat{v})R^{\lambda+\lambda'}v^{l+l'-\lambda-\lambda'}\end{aligned} \tag{9}$$

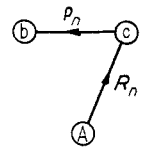
where

$$\sigma=\frac{3\gamma}{4}(\alpha+\tfrac{9}{16}\gamma),\qquad v=r_{3'4}+\sigma R_4,\qquad \gamma=\epsilon(\tfrac{4}{3}\sigma-\sigma^2),\qquad \epsilon=\frac{3\gamma}{4\sigma}$$

$$g_l=\left(\frac{4\pi[l]!}{[l-\lambda]![\lambda]!}\right)^{1/2},\qquad V_{b4}=V_0\exp(-\gamma\rho_{b4}{}^2),\qquad \alpha=\alpha_1+\alpha_2$$

and α_i is the oscillator parameter for the function

$$\Psi^*_{n_il_im_i}(r)=N_{l_i}r^{l_i}\exp(-\alpha_ir^2)\,Y_{l_im_i}(\hat{r}).$$



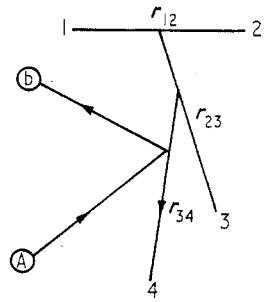


Figure 1. Vector diagram for the transfer process.

In deriving equation (7) the multipole expansion of spherical harmonics was used and the lowest value of $\Psi_{n_i l_i m_i}$, i.e. $n'_{3'4} = n_{3'4} = 0$, was considered.

This is the expression for the DWBA for the transfer of four nucleons. Taking now, as an illustration, the plane wave limit in equation (9), one has

$$\begin{aligned}\mathcal{T} = {} & 4v_0 \sum g_{l_{3'4}} g_{l'_{3'4}} (l'_{3'4} - \lambda'(m'_{3'4} - m_{\lambda'}), \lambda' m_{\lambda'} | l'_{3'4} m'_{3'4}) \\ & \times (l_{34} - \lambda(m_{3'4} - m_\lambda), \lambda m_\lambda | l_{3'4} m_{3'4}) (\lambda m_\lambda \lambda' - m_{\lambda'} | \Lambda m_\Lambda) (\Lambda m_\Lambda \nu m_\nu | L_4' M_4') \\ & \times (\lambda 0 \lambda' 0 | \Lambda 0)(\Lambda 0 \nu 0 | L_4' 0) \left(\frac{[\lambda][\lambda'][\nu]}{[L_4']} \right)^{1/2} (-)^{\lambda + \lambda' + m_\lambda + \frac{1}{2}(L_4 + L_4')} \\ & \times Y^*_{\nu m_\nu}(\hat{\boldsymbol{Q}}) Y_{L_4 M_4}(\hat{\boldsymbol{K}}) A_1 A_2 A_3 \end{aligned} \tag{10}$$

where

$$A_1 = \int \Psi_{N_4 L_4}(R) j_\nu(QR) R^{\lambda + \lambda' + 2} \exp(-\delta R^2)\, \mathrm{d}R$$

$$= \left(\frac{\pi}{2}\right)^{1/2} \mathcal{N}_{i_4} Q^\nu \frac{\frac{1}{2}\Gamma(\nu + \mu + 1)\, {}_1F_1\{\frac{1}{2}(\nu + \mu + 1), \nu + 1, -Q^2/4\eta\}}{2^{\nu+1} \eta \{(\nu + \mu + 1)/2\}\, \Gamma(\nu + 1)}$$

$$A_2 = \int \exp(-\epsilon v^2) v^{2(l_{34} - \lambda + 1)}\, \mathrm{d}v = \tfrac{1}{2} \epsilon^{-(l_{34} - \lambda + \frac{3}{2})} \Gamma(l_{34} - \lambda + \tfrac{3}{2})$$

$$A_3 = \int \psi_{N_4 L_4}(\rho) j_{L_4}(K\rho) \rho^2\, \mathrm{d}\rho = \left(\frac{\pi}{2}\right)^{1/2} \mathcal{N}_{L_4} (2a_2)^{-(L_4 + \frac{3}{2})} K^{L_4} \exp\left(-\frac{K^2}{4a_2}\right)$$

with

$$\boldsymbol{K} = \frac{b}{b+4} \boldsymbol{k}_i - \boldsymbol{k}_f, \qquad \boldsymbol{Q} = \boldsymbol{k}_i - \frac{A}{A+4} \boldsymbol{k}_f$$

$$\mu = \lambda + \lambda' + \tfrac{3}{2}, \qquad \eta = \delta + a_1.$$

a_1 and a_2 are the oscillator parameters for the functions $\psi_{N_{4'} L_{4'}}(R)$ and $\Psi_{N_4 L_4}(\rho)$ respectively. The lowest state, i.e. $N_4 = N_{4'} = 0$, is taken for both functions, and their normalization constants are denoted, respectively, by $\mathcal{N}_{L_{4'}}$ and $\mathcal{N}_{L_4}$.

4. A special example

The reaction ^{12}C(d, ^{6}Li)^{8}Be will now be considered, where, according to the shell model, the four transferred nucleons in ^{6}Li are $(1\,P_{3/2}{}^2, 1\,S_{1/2}{}^2)$ for which $\mathscr{L}_2' = 0$ and $\mathscr{L}_4' = 0$, and in ^{12}C are $(1\,P_{3/2}{}^4)$ for which $\mathscr{L}_4 = 0$ and $\mathscr{L}_2 = 0$.

For the pick-up mechanism, we replace $\boldsymbol{k}_i$ by $-\boldsymbol{k}_f$ in both $\boldsymbol{Q}$ and $\boldsymbol{K}$. In the present treatment we always consider the lowest oscillation state, i.e. with principal quantum number zero. One obtains finally for the transition amplitude

$$\begin{aligned} T_{fi} \propto {} & \sum (-1)^{l_{3'4} + L_4 + g + \Lambda + \lambda' + \frac{1}{2}(L_4 + L_4')} \frac{[g]}{([L_4][l_{34} - \lambda]!)} \\ & \times \left(\frac{[l_{34}][l_{34}'][\lambda][\lambda'][L_4][\Lambda][l_{34}]![l_{34}']!}{[\lambda][\lambda']!} \right)^{1/2} (\lambda 0 \lambda' 0 | \Lambda 0)(\Lambda 0 L_4 0 | L_4') \\ & \times \begin{Bmatrix} l_{34} & \Lambda & l_{34}' \\ L_4' & g & L_4 \end{Bmatrix} \begin{Bmatrix} l_{34} & \Lambda & l_{34}' \\ \lambda' & l_{34} - \lambda & \lambda \end{Bmatrix} \langle 0 L_4', 0 l_{34}'; 1 | 3, 1 | 0 L_{3'}, 0 l_4; 1 \rangle \\ & \times A_1 A_2 A_3 P_{L_4}(\widehat{\boldsymbol{Q}.\boldsymbol{K}}). \end{aligned} \tag{11}$$

The theoretical curve obtained by equation (11) for the plane wave limit is compared with the experimental data of Daehnick and Denes (1964). When the plane wave limit is applied to the experimental data only the forward peak is clearly defined. The second, smaller, peak does not appear in the plane wave cluster transfer model, as can be seen from figure 2. Better agreement is noticed with the distorted wave curve for the cluster, which is also shown in figure 2.

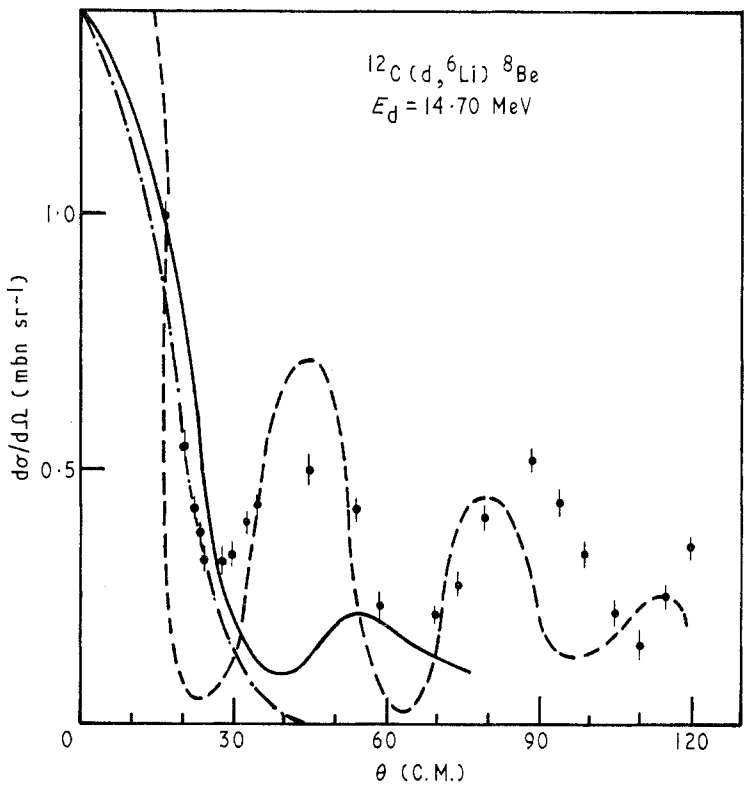


Figure 2. Angular distribution of ^{6}Li nuclei from the reaction ^{12}C(d, ^{6}Li)^{8}Be. The experimental data are those of Daehnick and Denes (1964). Full curve, plane wave calculations for four-nucleon transfer; broken curve, α-cluster transfer in the distorted wave Born approximation; chain curve, α-cluster transfer in plane wave Born approximation.

Owing to the short-comings of the plane wave calculations, it was necessary to perform calculations based on the diffraction model and these are reported in § 5.

5. Diffraction model

In this section, one may introduce the condition of strong absorption in both the initial and final channels in the evaluation of the amplitude given by equation (8) in § 3 (Henley 1965, Frahn 1966). This may seem a justifiable assumption owing to the composite nature of the incident and outgoing particles. Using a zero-range form for the potential $V(\boldsymbol{\rho}_{4b})$, i.e.

$$V(\boldsymbol{\rho}_{4b}) = V_0\delta(\boldsymbol{\rho}_4 - \tfrac{3}{4}\boldsymbol{r}_{34}). \tag{12}$$

Equation (8) may be written as

$$\mathcal{T} = \delta_{n_{12}n_{12}'}\delta_{l_{12}l_{12}'}\delta_{m_{12}m_{12}'}\ \delta_{n_{23}n_{23}'}\delta_{l_{23}l_{23}'}\delta_{m_{23}m_{23}'}4V_0$$
$$\langle\chi_{\boldsymbol{k}_f}^{(-)}\left(\boldsymbol{\rho}_4+\frac{A}{A+4}\boldsymbol{R}_4\right)\Psi_{n_{34}'l_{34}m_{34}'}(\tfrac{4}{3}\boldsymbol{\rho}_4)\Psi_{N_4'L_4'M_4'}(\boldsymbol{R}_4)|$$
$$\times\chi_{\boldsymbol{k}_i}^{(+)}\left(\frac{b}{b+4}\boldsymbol{\rho}_4+\boldsymbol{R}_4\right)\Psi_{n_{34}l_{34}m_{34}}(\tfrac{4}{3}\boldsymbol{\rho}_4)\Psi_{N_4L_4M_4}(\boldsymbol{\rho}_4)\rangle. \tag{13}$$

The following approximation for the function $\chi^{(\pm)}$ may be introduced:

$$\chi_{\boldsymbol{k}_f}^{(-)}\left(\boldsymbol{\rho}_4+\frac{A}{A+4}\boldsymbol{R}_4\right) = \exp(\boldsymbol{\rho}_4\cdot\nabla_{R_4})\chi_{\boldsymbol{k}_f}^{(-)}\left(\frac{A}{A+4}\boldsymbol{R}_4\right)$$
$$\simeq \exp\left(\mathrm{i}\frac{A}{A+4}\boldsymbol{k}_f\cdot\boldsymbol{\rho}_4\right)\chi_{\boldsymbol{k}_f}^{(-)}\left(\frac{A}{A+4}\boldsymbol{R}_4\right) \tag{14}$$

and

$$\chi_{\boldsymbol{k}_i}^{(+)}\left(\frac{b}{b+4}\boldsymbol{\rho}_4+\boldsymbol{R}_4\right) = \exp\left(\frac{b}{b+4}\boldsymbol{\rho}_4\cdot\nabla_{R_4}\right)\chi_{\boldsymbol{k}_i}^{(+)}(\boldsymbol{R}_4)$$
$$\simeq \exp\left(\mathrm{i}\frac{b}{b+4}\boldsymbol{k}_i\cdot\boldsymbol{\rho}_4\right)\chi_{\boldsymbol{k}_i}^{(+)}(\boldsymbol{R}_4)$$

where the operator ∇_{R_4} in the exponent is assumed to operate on the plane wave limit of the functions $\chi^{(\pm)}$. In this approximation equation (13) becomes

$$\mathcal{T} = 4V_0\int\exp(\mathrm{i}\boldsymbol{Q}'\cdot\boldsymbol{\rho}_4)\Psi_{n_{34}'l_{34}'m_{34}'}(\tfrac{4}{3}\boldsymbol{\rho}_4)\Psi_{n_{34}l_{34}m_{34}}(\tfrac{4}{3}\boldsymbol{\rho}_4)$$
$$\times\Psi_{N_4L_4M_4}(\boldsymbol{\rho}_4)\,\mathrm{d}\boldsymbol{\rho}_4\cdot B_{N'_4L'_4M'_4} \tag{15}$$

where

$$\boldsymbol{Q}' = \frac{b}{b+4}\boldsymbol{k}_i - \frac{A}{A+4}\boldsymbol{k}_f$$

and

$$B_{N'_4L'_4M'_4} = \int\chi_{\boldsymbol{k}_f}^{(-)*}\left(\frac{A}{A+4}\boldsymbol{R}_4\right)\Psi_{N'_4L'_4M'_4}(\boldsymbol{R}_4)\chi_{\boldsymbol{k}}^{(+)}(\boldsymbol{R}_4)\,\mathrm{d}\boldsymbol{R}_4. \tag{16}$$

Now, the distorted wave functions may be expanded in partial waves, the coordinate system being chosen such that $\boldsymbol{k}_i$ defines the z axis and $\boldsymbol{k}_i\times\boldsymbol{k}_f$ the y axis:

$$\chi_{\boldsymbol{k}_i}^{(+)}(\boldsymbol{R}_4) = \frac{\sqrt{(4\pi)}}{k_iR_4}\sum_{l'}\mathrm{i}^{l'}(2l'+1)^{1/2}\exp(\mathrm{i}\sigma_{l'}^{(\mathrm{i})})\mathscr{F}_{l'}^{(\mathrm{i})}(k_i, R_4)\,Y_{l_0'}(\hat{\boldsymbol{R}}_4)$$

$$\chi_{\boldsymbol{k}_f}^{(-)*}\left(\frac{A}{A+4}\boldsymbol{R}_4\right) = \frac{4\pi}{k_{f'}R_4}\sum\mathrm{i}^{-l''}\exp(\mathrm{i}\sigma_{l''}^{(\mathrm{f})})\mathscr{F}_{l''}^{(\mathrm{f})}(k_f', R_4)\,Y_{l''m''}(\theta, 0)\,Y_{l''m''}^{*}(\hat{\boldsymbol{R}}_4) \tag{17}$$

where θ is the scattering angle in the centre-of-mass system. Substituting equation (17) into equation (16), and integrating over $\hat{\boldsymbol{R}}_4$ yields:

$$B_{N_4'L_4'M_4'} = \frac{4\pi(2L_4'+1)^{1/2}}{k_ik_f'}\sum\mathrm{i}^{l'-l''-L_4'}(2l''+1)^{1/2}\exp\{\mathrm{i}(\sigma_{l'}^{(\mathrm{i})}+\sigma_{l''}^{(\mathrm{f})})\}(l''0, L0|l'0)$$
$$\times(l''M_4', L_4'-M_4'|l'0)\,Y_{l''-M_4'}(\theta, 0)R_{l''l'}^{N_4'L_4'}(k_i, k_f') \tag{18}$$

where

$$k_f' = \frac{A}{A+4} k_f$$

and where the radial integrals are defined by

$$R_{l''l'}^{N_4'L_4'}(k_i, k_f) = \int_0^\infty \mathcal{J}_{l''}^{(f)}(k_f', R_4) U_{N_4'L_4'}(R_4) \mathcal{J}_{l'}^{(i)}(k_i, R_4)\, dR_4. \qquad (19)$$

These radial integrals, for strongly absorbed particles, are peaked in a narrow region of l space around $l_0 = kR_0$, where R_0 is the interaction radius. As suggested by Sopkovich (1962), a DWBA radial integral may be expressed in terms of the same

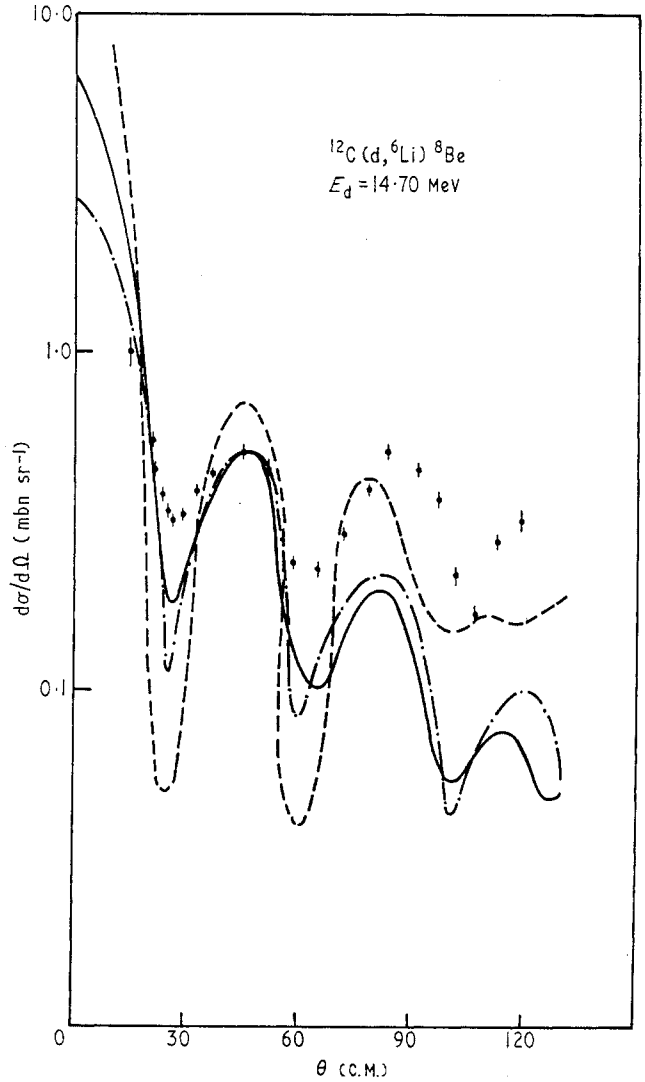


Figure 3. Angular distribution of ^{6}Li nuclei from the reaction ^{12}C(d, ^{6}Li)^{8}Be. Full curve, diffraction model calculations for four-nucleon transfer; broken curve, distorted wave Born approximation for the α-cluster transfer (Daehnick and Denes 1964); chain curve, diffraction model calculations for α-cluster transfer.

integral over the undistorted wave (except for the case of Coulomb distortion), multiplied by factors which are the square root of the elastic nuclear scattering functions in the initial and final channels, i.e.

$$R_{l''l'}^{N_4'L_4'}(k_i, k_f') = \{\eta_{l''}(k_f)\}^{1/2} I_{l''l'} \{\eta_{l'}(k_i)\}^{1/2} \tag{20}$$

where $I_{l''l'}$ is the radial integral for the case of no nuclear distortion. The factor $(\eta_{l'})^{1/2}$, for strong absorption, supresses contributions from partial waves below $l_0'' \simeq k_f R_0$, and $l_0' \simeq k_i R_0$. Hence one may use the asymptotic forms for the function $\not{f}_l$ and the bound-state wave function $U_{N_4'L_4'}$. Moreover, if L_4 is small compared with the cut-off values of $l_0'' = l_0'$, one may write $I_{l''l'}$ as I, with $l = \frac{1}{2}(l''+l')$.

Using, now, the expression obtained by Dar (1965, 1967) for the radial integral I and following the same procedure used by Potgieter and Frahn (1966) and Frahn and Sharaf (1969) for evaluating the sum over l in equation (18), one finds, finally, for $B_{N_4'L_4'M_4'}$

$$B_{N_4'L_4'M_4'} = \frac{(2\pi)^2}{k_i k_f'} \frac{1}{\kappa\sqrt{\delta}} \exp(2i\sigma_{l_0}) \exp[-\{\gamma + \nu\phi + \delta(l_0+\tfrac{1}{2})\}] Y_{L_4M_4}(\tfrac{1}{2}\pi, 0) S_{l_0}^{M_4} \tag{21}$$

where

$$S_{l_0}^{M_4} = \frac{1}{2\sqrt{(2\pi)}} (-1)^{\frac{1}{2}(M_4' - |M_4'|)} (l_0+\tfrac{1}{2})^{1/2} \left(\frac{\theta}{\sin\theta}\right)^{1/2}$$
$$\times [(H_+ + H_-) J_{|M_4'|}\{(l_0+\tfrac{1}{2})\theta\} - i(H_+ - H_-) J_{|M_4'|-1}\{(l_0+\tfrac{1}{2})\theta\}] \tag{22}$$
$$H_\pm = \frac{i\{1+\mu(\theta_C \pm \theta + i\delta)\}\pi\Delta}{\sinh\{(\theta_C \pm \theta + i\delta)\Delta\pi\}} - \frac{1}{2} \frac{\lambda(\theta_C \pm \theta + i\delta)\Delta\pi}{\sinh\{\pi\Delta(\theta_C \pm \theta + i\delta)\}}$$

and θ_C is the Coulomb angle.

In evaluating this expression, η_l is taken as

$$\eta_l = \left(1 + i\mu\frac{\partial}{\partial l}\right) g_l, \qquad g_l = \left\{1 + \exp\left(\frac{l-l_0}{\Delta}\right)\right\}^{-1}.$$

If one adopts this procedure for the calculation of the reaction ^{12}C(d, ^{6}Li)^{8}Be and applies the selection rules used in § 4, the angular distribution obtained for both four-nucleon and α-cluster transfer models is as shown in figure 3. Agreement with the data is very satisfactory up to 60°.

6. Discussion

From equation (9), one may notice that the nucleons 1 and 2 are transferred to the target nucleus in the same relative l state as they were in the projectile; the relative motion between the centre of mass of the first two nucleons and the third nucleon is also the same in the two states.

This means that the two states of the four transferred nucleons differ only in the relative motion between the centre of mass of the first three nucleons and the fourth nucleon. This is also valid for the general case of transfer of n nucleons, where the relative l state between the centre of mass of the transferred $n-1$ nucleons and the nth nucleon will be the only state changed. In the case of the cluster transfer reactions (Abul-Magd *et al.* 1965), the interaction responsible for the transition V_{cb} is taken to depend on the distance between the centre of mass of the four-nucleon cluster and that of the outgoing particle, while in the present treatment the interaction is taken as a sum of the interaction of the outgoing particle with each of the four captured nucleons separately. The residual interaction $V_{1b}+V_{2b}+V_{3b}+V_{4b}-V_{cb}$ might cause the cluster composed of the four captured nucleons to suffer internal excitation in the process of its transfer.

The effect of this internal excitation may explain the deviation between the four-nucleon transfer results and those deduced for the case of cluster transfer, which can be seen in both figure 2 and figure 3. Although the difference between the four-nucleon transfer and the cluster transfer is small in the diffraction model when using the approximations introduced above, one may expect these differences to be small in the case of multi-nucleon transfer. Better agreement with experimental data may be obtained with the diffraction model formula either by considering higher-energy data (and consequently large l_0) or using a more accurate method of evaluating the radial integrals. But, in any case, the diffraction model curve gives the exact trend of the angular distribution, and the diffraction curve for the cluster transfer is comparable with the DWBA curve.

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